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Logiciel Maple

Méthodes & TPs

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TPs destinés à la promotion 3iem année Licence physique des matériaux semestre 5 (L3/S5)

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Logiciel Maple

Avant-propos

Cet ouvrage est une collection des exemples mathématiques et méthodes numériques appliquée à la physique programmés dans le code de Maple. Elle est destinée aux étudiants de 3^{iem} année Licence physique des matériaux semestre 5 (L3/S5).

Table des matières

Introduction

Maple is a computer algebra software: it can process numerical data (integer, reals, complexes, etc.) of arbitrary precision and also symbolic data (polynomials, expressions, etc.). This software is also endowed with graphics capabilities. We define the following characters:

1. Package
2. Instruction
3. Result

Exemple

> $with(LinearAlgebra):$ → **Package**

> $M := Matrix(2, 2, [1, 2, 3, 4])$ → **Instruction**

$$M := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$ → **Result**

Chapter 1 Manipulation of Basic examples

1.1 Functions

> $f := (x) \rightarrow \frac{1}{\exp(x) + 1}$ *#commentaire*

$$f := x \rightarrow \frac{1}{e^{x} + 1}$$

> $f(2)$

$$\frac{1}{e^{2} + 1}$$

> $evalf(\%)$

$$0.1192029220$$

> $diff(f(x), x)$

$$-\frac{e^{x}}{\left(e^{x} + 1\right)^{2}}$$

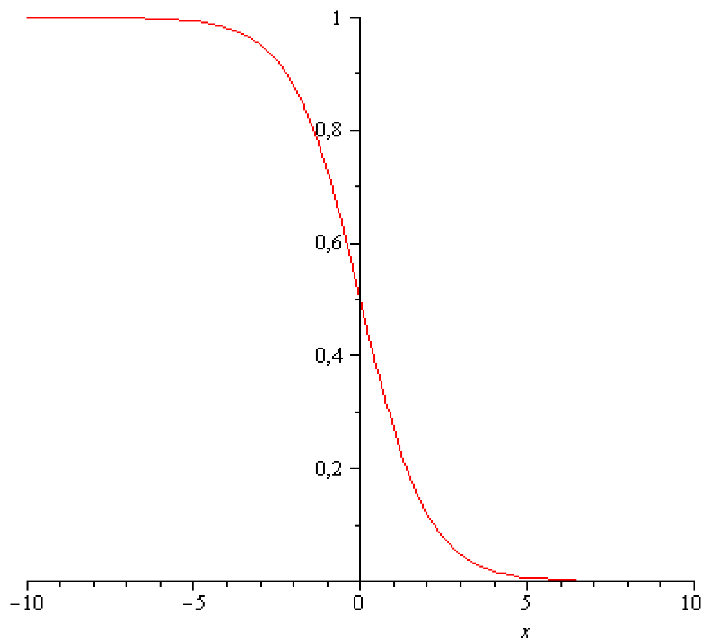
> $Int(f(x), x = 2 ..3)$

$$\int_{2}^{3} \frac{1}{e^{x} + 1} \, dx$$

> $int(f(x), x = 2 ..3)$

$$\ln\left(e^{2} + 1\right) + 1 - \ln\left(e^{3} + 1\right)$$

> $plot(f(x), x = -10 ..10)$



> $solve(f(x) = 0.20)$

$$1.386294361$$

1.2 Matrix

> $with(LinearAlgebra) : with(linalg) :$

> $M := Matrix(2, 3, [1, 2, 3, 4, 5, 6])$

$$M := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

> $M(1, 3)$

$$3$$

> $op(M)$

$$2, 3, \{(1, 1) = 1, (1, 2) = 2, (1, 3) = 3, (2, 1) = 4, (2, 2) = 5, (2, 3) = 6\}, datatype = anything, storage = rectangular, order = Fortran\_order, shape = [\ ]$$

> $N := Matrix(3, 2, [1, 2, 3, 4, 5, 6])$

$$N := \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

> $A := M.N; B := N.M$

$$A := \begin{bmatrix} 22 & 28 \\ 49 & 64 \end{bmatrix}$$

$$B := \begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 29 & 40 & 51 \end{bmatrix}$$

> $Determinant(A)$

$$36$$

> $MatrixInverse(A)$

$$\begin{bmatrix} \frac{16}{9} & -\frac{7}{9} \\ -\frac{49}{36} & \frac{11}{18} \end{bmatrix}$$

> $Eigenvectors(A)$

$$\begin{bmatrix} 43 + 7\sqrt{37} \\ 43 - 7\sqrt{37} \end{bmatrix}, \begin{bmatrix} \frac{28}{21 + 7\sqrt{37}} & \frac{28}{21 - 7\sqrt{37}} \\ 1 & 1 \end{bmatrix}$$

1.3 Tests, Loops, and Procedures

> $product((z-i), i = 1..4)$

$$(z-1)\ (z-2)\ (z-3)\ (z-4)$$

> $sum((z[i]-i), i = 1..4)$

$$z_1 - 10 + z_2 + z_3 + z_4$$

> $seq(z-i, i = 1..4)$

$$z-1, z-2, z-3, z-4$$

> $s := 1;$
for i **from** 1 **to** 4 **do** $s := 2 + s \cdot s$ **end do**

$$s := 1$$

$$s := 3$$

$$s := 11$$

$$s := 123$$

$$s := 15131$$

> $a := 10$

$$a := 10$$

> **if** $a = 10$ **then** $a \cdot a$ **else** $a - 1$ **fi**

$$100$$

>
$Root$:= **proc**(x);
if $x < 0$ **then return**$(I \cdot \mathrm{sqrt}(-x))$
else return $(\mathrm{sqrt}(x))$
fi
end proc

```
Root := proc(x)
    if x < 0 then
        return I * sqrt( - x)
    else
        return sqrt(x)
    end if
end proc
```

```
> Root( -9)
```

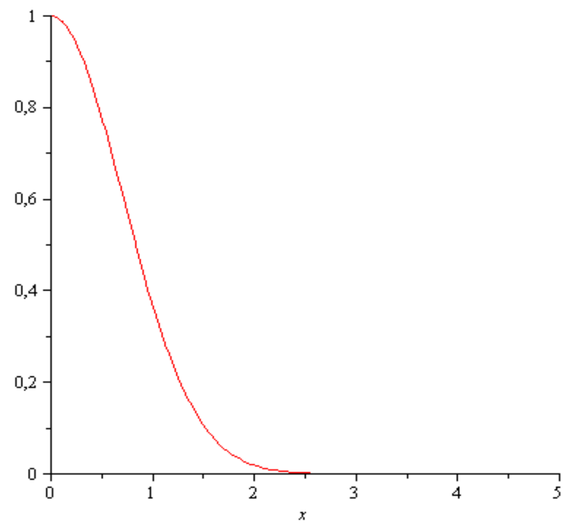
$$3\,I$$

Chapter 2 Manipulation of function problems

2.1 Simpson and trapezoid methods

> $with(Student[Calculus1])$:

> $plot\left(\exp(-x^2), x = 0..5\right)$



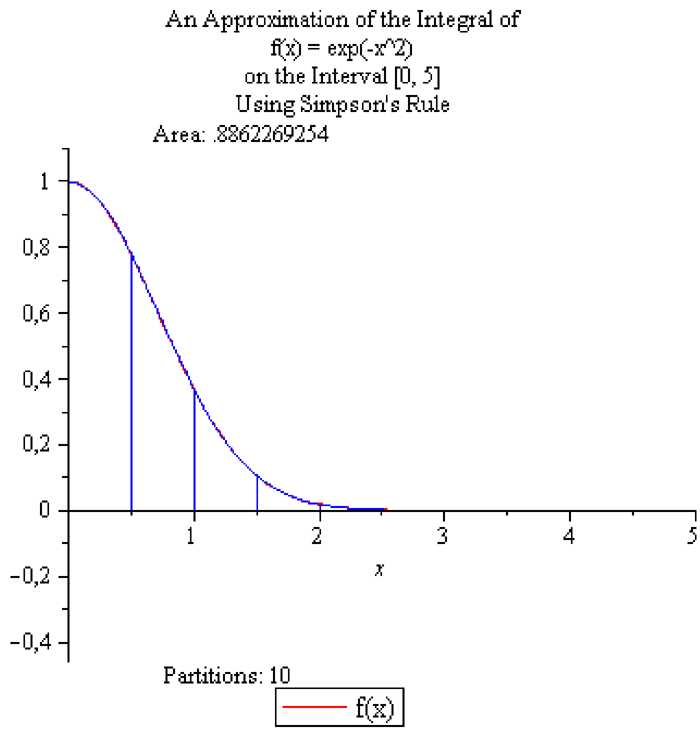
> $int\left(\exp(-x^2), x = 0..5\right)$

$$\frac{1}{2}\,\mathrm{erf}(5)\,\sqrt{\pi}$$

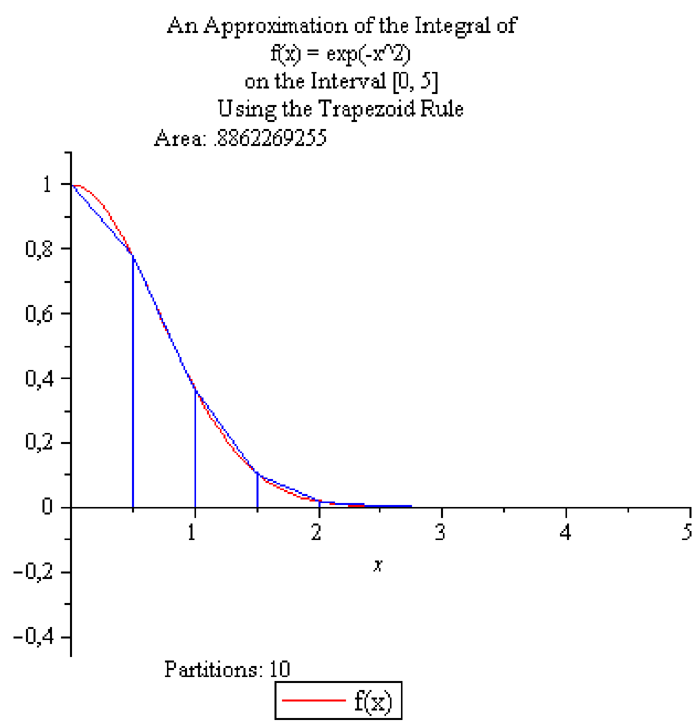
> $evalf(\%)$

$$0.8862269255$$

> $ApproximateInt\left(\exp(-x^2), 0..5, method = simpson, partition = 10, output = animation\right)$



$ApproximateInt\left(\exp(-x^2), 0..5, method = trapezoid, output = animation\right)$



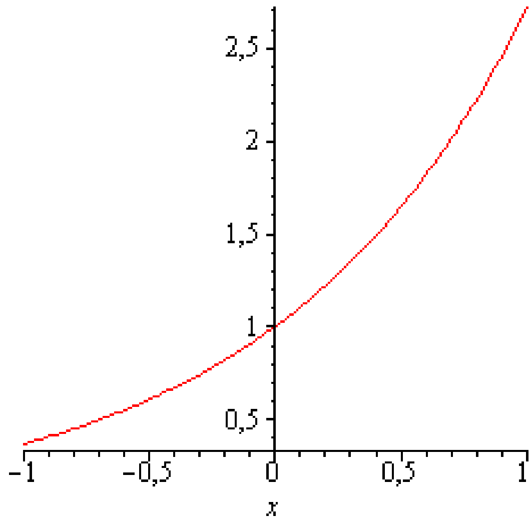
2.2 Series development

2.2.1 Calculation of the derivative of a function

> $f := (x) \rightarrow \exp(x)$

$$f := x \rightarrow e^{x}$$

> $plot(f(x), x=-1..1)$



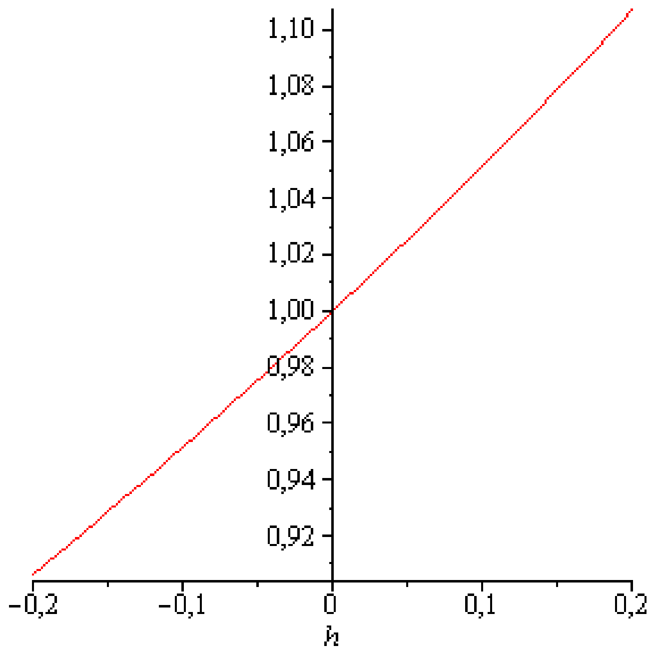
> $\dfrac{(f(x+h)-f(x))}{h}$

$$\frac{e^{x+h}-e^{x}}{h}$$

> $\left(\dfrac{(\exp(h)-1)}{h}, h=0\right)$

$$\frac{e^{h}-1}{h}, h=0$$

> $plot\left(\dfrac{e^{h}-1}{h}, h=-0.2..0.2\right)$



2.2.2 Series development in different intervals

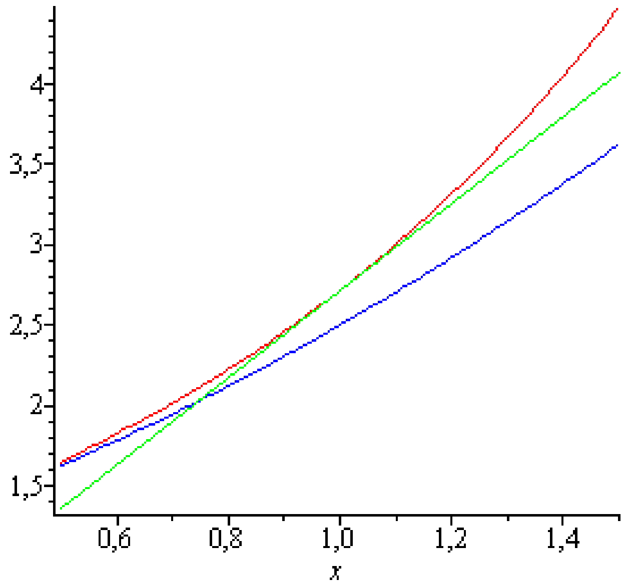
> $Df1 := (x) \rightarrow series(\exp(x), x = 0, 3);$
> $Df2 := (x) \rightarrow series(\exp(x), x = 1, 2)$

$$Df1 := x \rightarrow series\left(e^x, x = 0, 3\right)$$
$$Df2 := x \rightarrow series\left(e^x, x = 1, 2\right)$$

> $P1 := (x) \rightarrow convert(Df1(x), polynom);$
> $P2 := (x) \rightarrow convert(Df2(x), polynom)$

$$P1 := x \rightarrow convert(Df1(x), polynom)$$
$$P2 := x \rightarrow convert(Df2(x), polynom)$$

> $plot([f(x), P1(x), P2(x)], x = 0.5 ..1.5, color = [red, blue, green])$



Exercice: Do the same think for the functions ln, and sin

2.3 Interpolation & approximation

> $with(orthopoly), with(LinearAlgebra), with(linalg):$

2.3.1 Orthogonal polynomiales

(a) Legendre polynomial

> $pol := (n, x) \rightarrow \frac{1}{2^n \cdot n!} \cdot diff\left((x^2-1)^n, [x\$n]\right)$# *Legendre polynomial*

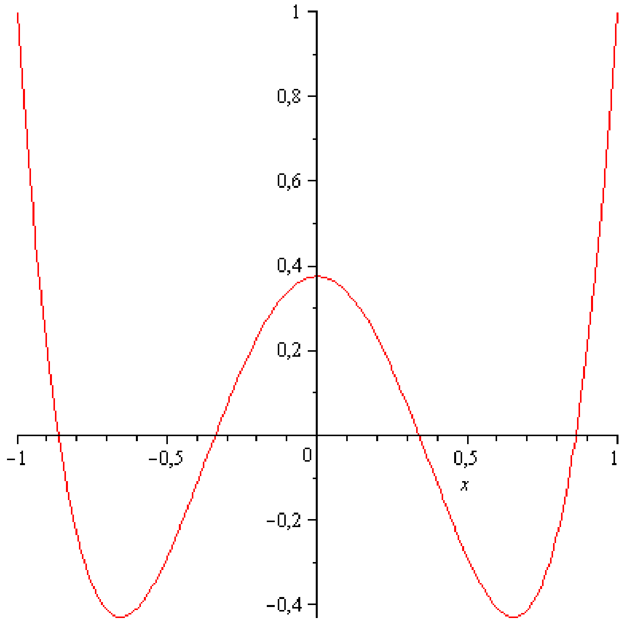
$$pol := (n, x) \rightarrow \frac{\frac{\partial^n}{\partial x^n} (x^2-1)^n}{2^n\, n!}$$

> $expand(pol(4, x)); P(4, x)$

$$\frac{35}{8} x^4 - \frac{15}{4} x^2 + \frac{3}{8}$$

$$\frac{35}{8} x^4 - \frac{15}{4} x^2 + \frac{3}{8}$$

> $plot(pol(4, x), x = -1..1)$



> $int(P(2, x) \cdot P(2, x), x = -1..1)$

$$\frac{2}{5}$$

> $P(4, 1)$

$$1$$

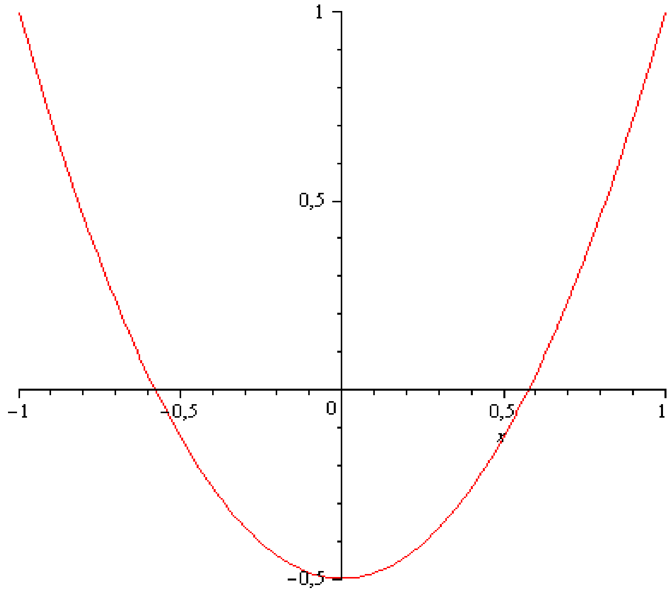
> $int(P(2, x), x = -0.5..0.5)$

$$-0.3750000000$$

> $P(3, x)$

$$\frac{5}{2}x^3 - \frac{3}{2}x$$

> $plot(P(2, x), x = -1..1)$



> $int(P(2, x), x = -0.5 ..0.5)$

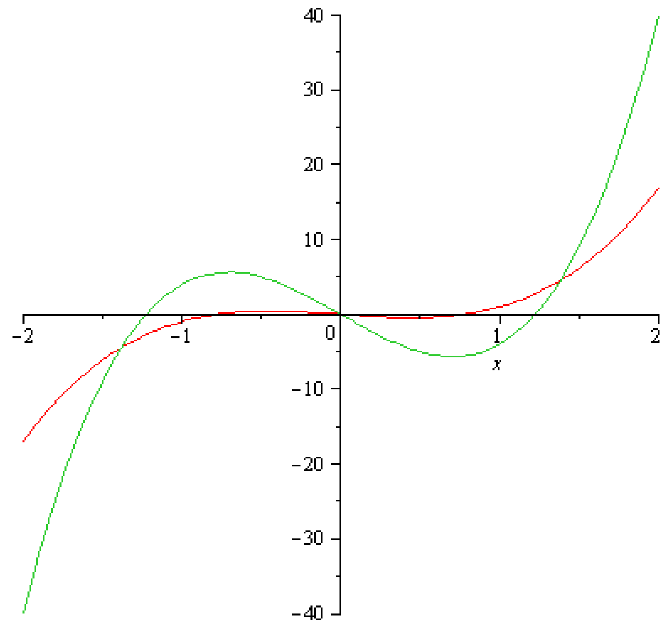
$$-0.3750000000$$

(b) Hermite polynomial

> $H(3, x)$

$$8\,x^3 - 12\,x$$

> $plot([P(3, x), H(3, x)], x = -2 ..2)$



> $int(H(3, x), x = -1 ..1)$

$$0$$

> $int(H(3, x) \cdot P(3, x), x = -1..1)$

$$\frac{32}{35}$$

> $int(H(5, x) \cdot P(3, x), x = -1..1)$

$$-\frac{128}{9}$$

2.3.2 Interpolation

> $a := \langle 1, 2.21, 3.36, 4, 5.1, 1\rangle; b := \langle 2.45, 8.65, 10.25, 7.35, 12\rangle$

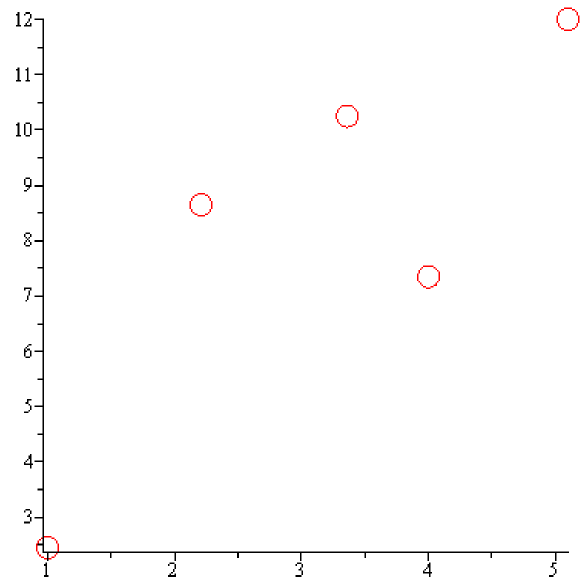
$$a := \begin{bmatrix} 1 \\ 2.21 \\ 3.36 \\ 4 \\ 5.1 \\ 1 \end{bmatrix}$$

$$b := \begin{bmatrix} 2.45 \\ 8.65 \\ 10.25 \\ 7.35 \\ 12 \end{bmatrix}$$

> $N := 5$

$$N := 5$$

> $plot(\langle\langle seq(a[i], i = 1..N)\rangle | \langle seq(b[i], i = 1..N)\rangle\rangle, style = point, symbol = circle, symbolsize = 30, color = red)$



> $A := Matrix\left(N, N, \left[seq\left(seq\left(a[k]^j, j = 0 \,..N - 1\right), k = 1 \,..N\right)\right]\right);$

$$A := [[1, 1, 1, 1, 1], [1.0, 2.21, 4.8841, 10.793861, 23.85443281], [1.0, 3.36, 11.2896, 37.933056, 127.4550682], [1, 4, 16, 64, 256], [1.0, 5.1, 26.01, 132.651, 676.5201]]$$

> $C := A^{-1}.b$

$$C := \begin{bmatrix} 23.1880167881767854 \\ -47.4482434723279454 \\ 35.3674289184687965 \\ -9.50166597078618302 \\ 0.844463736468555215 \end{bmatrix}$$

> **for** i **from** 0 **to** $N-1$ **do** $\text{alpha}[i] := C(i+1)$ **end do**

$$\alpha_0 := 23.1880167881767854$$

$$\alpha_1 := -47.4482434723279454$$

$$\alpha_2 := 35.3674289184687965$$

$$\alpha_3 := -9.50166597078618302$$

$$\alpha_4 := 0.844463736468555215$$

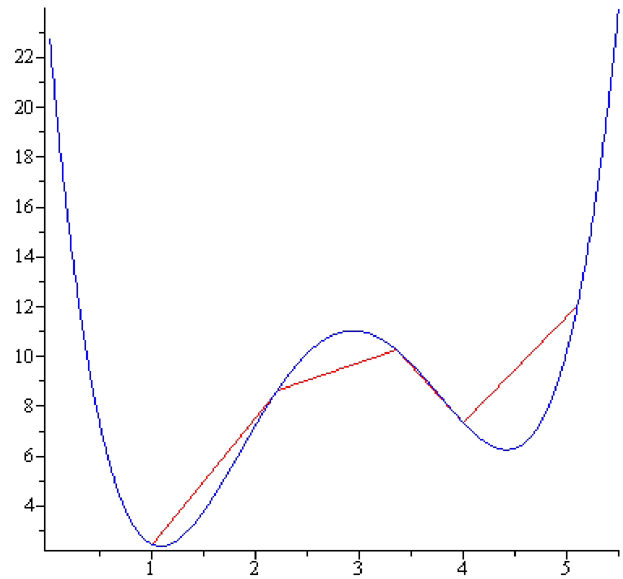
> $P := (x) \rightarrow add\left(\text{alpha}[i] \cdot (x)^i, i = 0..N-1\right)$

$$P := x \rightarrow add\left(\alpha_i\, x^i, i = 0..N-1\right)$$

> $P(x)$

$$23.18801679 - 47.4482434723279454\, x + 35.3674289184687965\, x^2 - 9.50166597078618302\, x^3 + 0.844463736468555215\, x^4$$

> $plot\left(\left[\langle\langle seq(a[i], i = 1 ..N)\rangle | \langle seq(b[i], i = 1 ..N)\rangle\rangle, \left\langle\left\langle seq\left(\frac{i}{100}, i = 1 .. 110\, N\right)\right\rangle \middle| \left\langle seq\left(P\left(\frac{i}{100}\right), i = 1 .. 110\, N\right)\right\rangle\right\rangle\right], linestyle = [solid, solid], color = [red, blue]\right)$



2.3.3 Least squares

> **for** i **from** 1 **to** 24 **do** $x[i] := i$ **end do**:

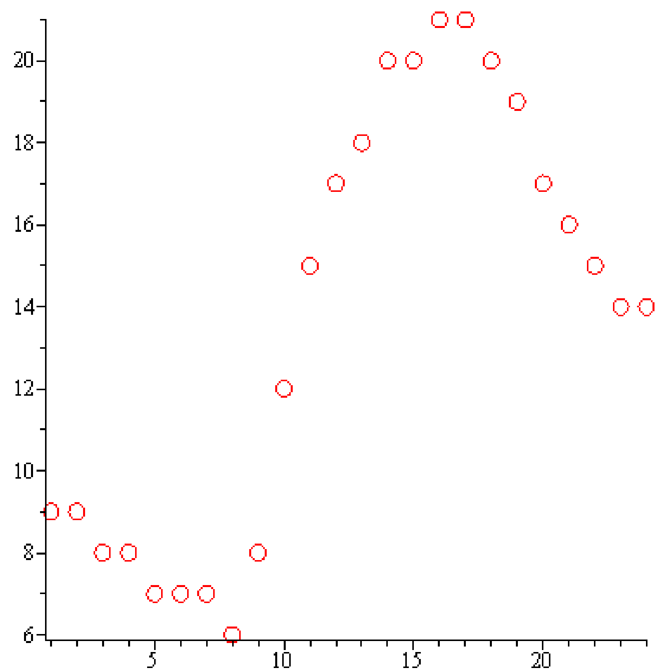
> $y := \langle 9, 9, 8, 8, 7, 7, 7, 6, 8, 12, 15, 17, 18, 20, 20, 21, 21, 20, 19, 17, 16, 15, 14, 14\rangle$

$$y := \begin{bmatrix} \text{1 .. 24 Vector}_{column} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix}$$

> $N := 16$

$$N := 16$$

> $plot(\langle\langle seq(x[i], i = 1..24)\rangle | \langle seq(y[i], i = 1..24)\rangle\rangle, style = point, symbol = circle, symbolsize = 20, color = red)$



> $f := (x) \rightarrow a0 + a1 \cdot x + a2 \cdot x^2 + a3 \cdot x^3$

$$f := x \rightarrow a0 + a1\, x + a2\, x^2 + a3\, x^3$$

> $S := (a0, a1, a2, a3) \rightarrow add\big((f(x[i]) - y[i])^2, i = 1..N\big)$

$$S := (a0, a1, a2, a3) \rightarrow add\Big(\big(f\big(x_i\big) - y_i\big)^2, i = 1..N \Big)$$

> $S(a0, a1, a2, a3)$

$$(a0 + a1 + a2 + a3 - 9)^2 + (a0 + 2\,a1 + 4\,a2 + 8\,a3 - 9)^2 + (a0 + 3\,a1 + 9\,a2 + 27\,a3 - 8)^2 + (a0 + 4\,a1 + 16\,a2 + 64\,a3 - 8)^2 + (a0 + 5\,a1 + 25\,a2 + 125\,a3 - 7)^2 + (a0 + 6\,a1 + 36\,a2 + 216\,a3 - 7)^2 + (a0 + 7\,a1 + 49\,a2 + 343\,a3 - 7)^2 + (a0 + 8\,a1 + 64\,a2 + 512\,a3 - 6)^2 + (a0 + 9\,a1 + 81\,a2 + 729\,a3 - 8)^2 + (a0 + 10\,a1 + 100\,a2 + 1000\,a3 - 12)^2 + (a0 + 11\,a1 + 121\,a2 + 1331\,a3 - 15)^2 + (a0 + 12\,a1 + 144\,a2 + 1728\,a3 - 17)^2 + (a0 + 13\,a1 + 169\,a2 + 2197\,a3 - 18)^2 + (a0 + 14\,a1 + 196\,a2 + 2744\,a3 - 20)^2 + (a0 + 15\,a1 + 225\,a2 + 3375\,a3 - 20)^2 + (a0 + 16\,a1 + 256\,a2 + 4096\,a3 - 21)^2$$

> $Sol := evalf(solve(\{diff(S(a0, a1, a2, a3), a0) = 0, diff(S(a0, a1, a2, a3), a1) = 0, diff(S(a0, a1, a2, a3), a2) = 0, diff(S(a0, a1, a2, a3), a3) = 0\}, \{a0, a1, a2, a3\}))$

$$Sol := \{a0 = 13.62637363, a1 = -3.456720610, a2 = 0.4635400719, a3 = -0.01348204599\}$$

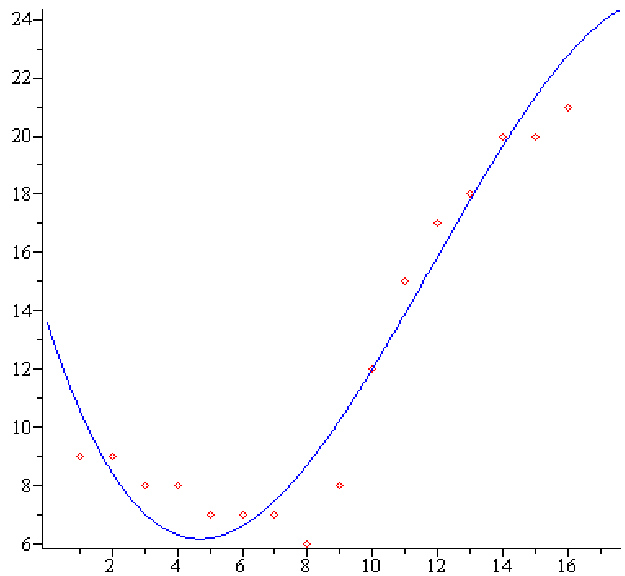
> $Sol[1]$

$$a0 = 13.62637363$$

$> f := (x) \rightarrow 13.63 - 3.46\,x + 0.46\,x^2 - 0.013 \cdot x^3$

$$f := x \rightarrow 13.63 + (-1) \cdot 3.46\,x + 0.46\,x^2 + (-1) \cdot 0.013\,x^3$$

$> plot\left(\left[\langle\langle seq(x[i], i = 1..N)\rangle | \langle seq(y[i], i = 1..N)\rangle\rangle, \left\langle\left\langle seq\left(\frac{i}{100}, i = 1..110\,N\right)\right\rangle \middle| \left\langle seq\left(f\left(\frac{i}{100}\right), i = 1..110\,N\right)\right\rangle\right\rangle\right], linestyle = [dot, solid], style = [point, line], color = [red, blue]\right)$



2.4 Fourier series

2.4.1 Definition of the coefficients

> $a := (n) \rightarrow \frac{2}{T} \cdot int\left(f(x) \cdot \cos(n \cdot w \cdot x), x = -\frac{T}{2} .. \frac{T}{2}\right)$ #$n \geq 0$

$$a := n \rightarrow \frac{2 \left(\int_{-\frac{1}{2} T}^{\frac{1}{2} T} f(x) \cos(n\, w\, x) \, dx \right)}{T}$$

> $b := (n) \rightarrow \frac{2}{T} \cdot int\left(f(x) \cdot \sin(n \cdot w \cdot x), x = -\frac{T}{2} .. \frac{T}{2}\right)$ # $n \geq 1$

$$b := n \rightarrow \frac{2 \left(\int_{-\frac{1}{2} T}^{\frac{1}{2} T} f(x) \sin(n\, w\, x) \, dx \right)}{T}$$

> $F := (N, x) \rightarrow \frac{a(0)}{2} + sum(a(n) \cdot \cos(n \cdot w \cdot x) + b(n) \cdot \sin(n \cdot w \cdot x), n = 1 .. N)$

$$F := (N, x) \rightarrow \frac{1}{2} a(0) + \sum_{n=1}^{N} \left(a(n) \cos(n\, w\, x) + b(n) \sin(n\, w\, x) \right)$$

> $T := T; w := \frac{2 \text{ Pi}}{T}$

$$T := T$$

$$w := \frac{2\pi}{T}$$

2.4.2 Example

> $f1 := (x) \rightarrow \text{Pi} + x; f2 := (x) \rightarrow \text{Pi} - x$

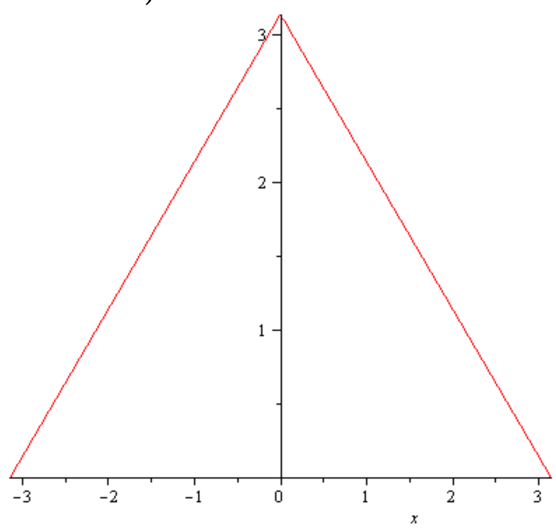
$$f1 := x \rightarrow \pi + x$$

$$f2 := x \rightarrow \pi - x$$

> $f := (x) \rightarrow piecewise(-Pi \le x \le 0, f1(x), 0 \le x \le Pi, f2(x))$

$$f := x \rightarrow piecewise\left(-\pi \le x \textbf{ and } x \le 0, f1(x), 0 \le x \textbf{ and } x \le \pi, f2(x)\right)$$

> $plot(f(x), x = -\text{Pi}..\text{Pi})$



> $g := (n, x) \rightarrow piecewise(-2\,\text{Pi} + (2\,n + 1) \cdot \text{Pi} \le x \le 2\,n \cdot Pi, f1(x) - 2\,n \cdot \text{Pi}, 2\,n \cdot \text{Pi} \le x \le (2\,n + 1) \cdot \text{Pi}, \text{f2}(x) + 2\,n \cdot \text{Pi})$

$$g := (n, x) \rightarrow \mathit{piecewise}\big(-2\pi + (2n + 1)\pi \le x \textbf{ and } x \le 2n\pi, \mathit{f1}(x) - 2n\pi, 2n\pi \le x \textbf{ and } x \le (2n + 1)\pi, \mathit{f2}(x) + 2n\pi\big)$$

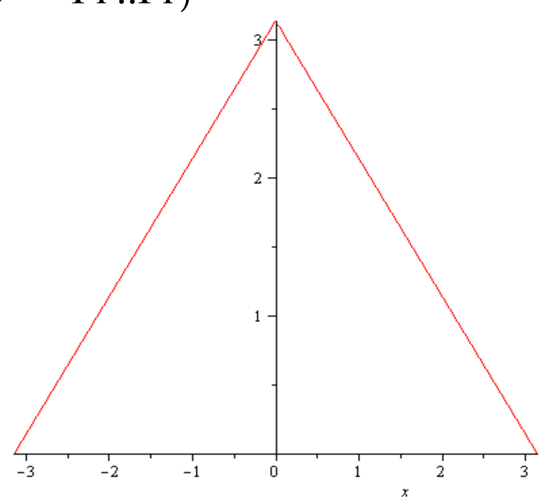
> $g(0, x); g(1, x); g(2, x)$

$$\begin{cases} x + \pi & -\pi \le x \textbf{ and } x \le 0 \\ \pi - x & 0 \le x \textbf{ and } x \le \pi \end{cases}$$

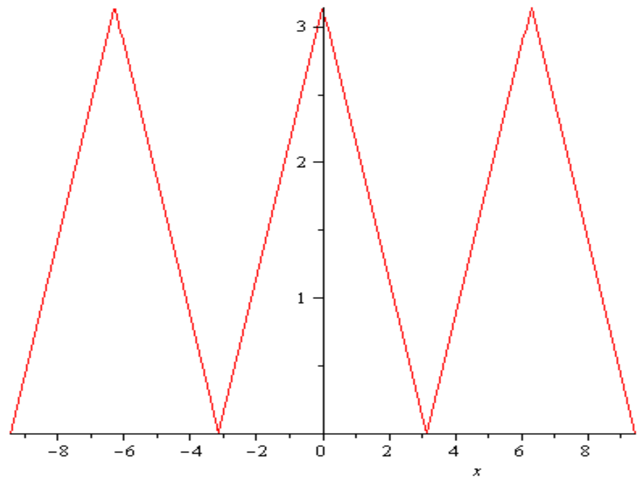
$$\begin{cases} -\pi + x & \pi \le x \textbf{ and } x \le 2\pi \\ 3\pi - x & 2\pi \le x \textbf{ and } x \le 3\pi \end{cases}$$

$$\begin{cases} -3\pi + x & 3\pi \le x \textbf{ and } x \le 4\pi \\ 5\pi - x & 4\pi \le x \textbf{ and } x \le 5\pi \end{cases}$$

> $\mathit{plot}(g(0, x), x = -\mathrm{Pi}\,..\mathrm{Pi})$



> $\mathit{plot}\big([g(-1, x), g(0, x), g(1, x)], x = -3\,\mathrm{Pi}\,..3\pi, \mathit{color} = [\mathit{red}]\big)$



> $T := 2\text{ Pi}; w;$

$$T := 2\pi$$

$$1$$

> $a(n); a(0)$

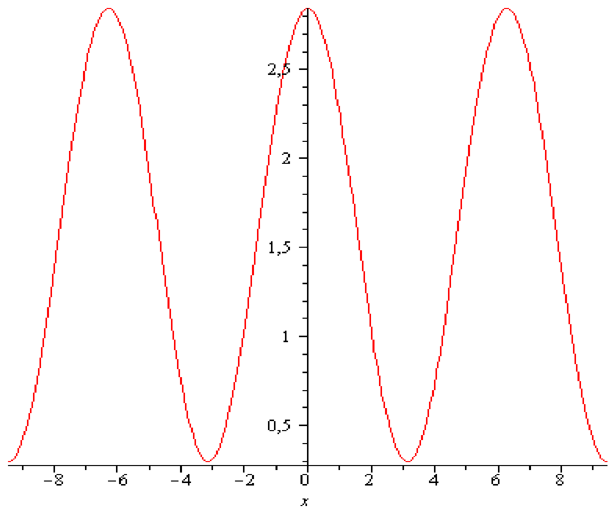
$$-\frac{2\left(\cos(\pi n) - 1\right)}{\pi n^2}$$

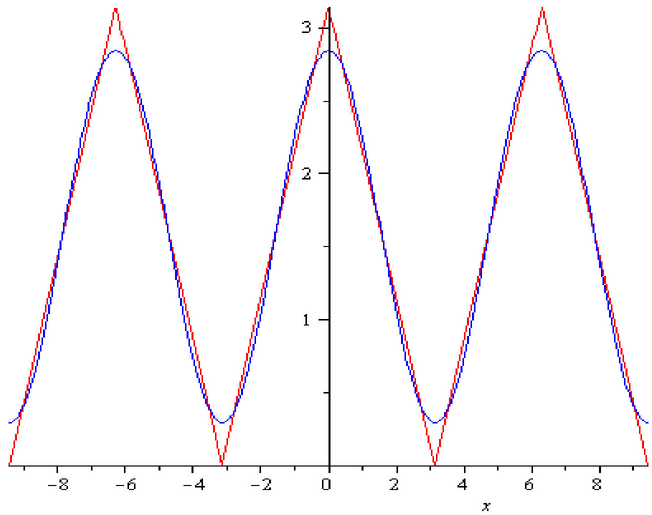
$$\pi$$

> $b(n)$

$$0$$

> $plot(F(2, x), x = -3\text{ Pi}..3\text{ Pi});$
$plot\left([g(-1, x), g(0, x), g(1, x), F(2, x)], x = -3\text{ Pi}..3\pi, color = [red, red, red, blue]\right)$





2.4.3 Value of π

> $f(x) = F(5, x)$

$$\begin{cases} x + \pi & -\pi \le x \textbf{ and } x \le 0 \\ \pi - x & 0 \le x \textbf{ and } x \le \pi \end{cases} = \frac{1}{2}\pi + \frac{4\cos(x)}{\pi} + \frac{4}{9}\frac{\cos(3x)}{\pi} + \frac{4}{25}\frac{\cos(5x)}{\pi}$$

> $EQ := f(0) = F(15, 0)$

$$EQ := \pi = \frac{1}{2}\pi + \frac{1951933472}{405810405\,\pi}$$

> $evalf\left(solve\left(vp = \frac{1}{2}vp + \frac{1951933472}{405810405\,vp}, vp\right)\right)$

$$3.101600904, \ -3.101600904$$

Chapter 3 Manipulation of equations

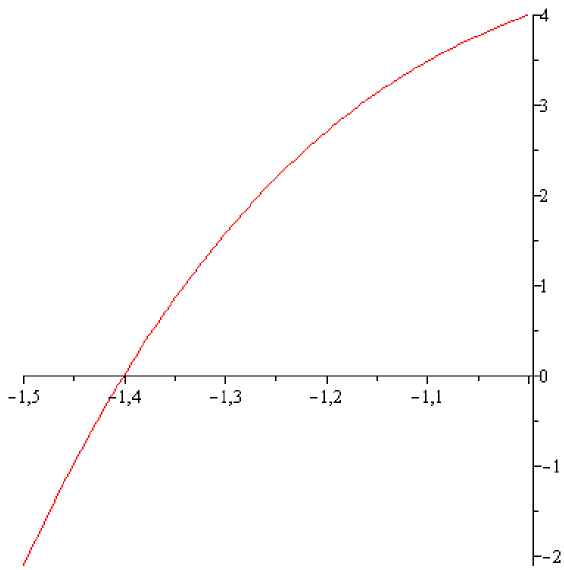
3.1 Zero of function

3.1.1 Definition of an equation

> $f := (x) \rightarrow x^5 - x + 4$

$$f := x \rightarrow x^5 - x + 4$$

> $plot(f(x), x = -1.5 .. -1)$



> $f(-1.5)$

$$-2.09375$$

3.1.2 Numerical resolution

> **for** i **from** 1 **while** $f\left(-1.5 + \frac{i}{100}\right) < 0$

do $x[i] := -1.5 + \frac{i}{100}; f(x[i])$ **end**

do

$$x_1 := -1.490000000$$

$$-1.853977575$$

$$x_2 := -1.480000000$$

$$-1.620821197$$

$$x_3 := -1.470000000$$

$$-1.394148551$$

$$x_4 := -1.460000000$$

$$-1.173829098$$

$$x_5 := -1.450000000$$

$$-0.959734062$$

$$x_6 := -1.440000000$$

$$-0.751736422$$

$$x_7 := -1.430000000$$

$$-0.549710894$$

$$x_8 := -1.420000000$$

$$-0.353533923$$

$$x_9 := -1.410000000$$

$$-0.163083670$$

> **for** i **from** 1 **while** $f\left(x[9] + \frac{i}{1000}\right) < 0$

do $x[i] := x[9] + \frac{i}{1000}; f(x[i])$

end do

$$x_1 := -1.409000000$$

$$-0.144348974$$

$$x_2 := -1.408000000$$

$$-0.125670224$$

$$x_3 := -1.407000000$$

$$-0.107047300$$

$$x_4 := -1.406000000$$

$$-0.088480083$$

$$x_5 := -1.405000000$$

$$-0.069968454$$

$$x_6 := -1.404000000$$

$$-0.051512296$$

$$x_7 := -1.403000000$$

$$-0.033111490$$

$$x_8 := -1.402000000$$

$$-0.014765917$$

> **for** i **from** 1 **while** $f\left(x[8] + \frac{i}{10000}\right) < 0$ **do** $x[i] := x[8] + \frac{i}{10000}$; $f(x[i])$ **end do**

$$x_1 := -1.401900000$$

$$-0.012934393$$

$$x_2 := -1.401800000$$

$$-0.011103420$$

$$x_3 := -1.401700000$$

$$-0.009272998$$

$$x_4 := -1.401600000$$

$$-0.007443127$$

$$x_5 := -1.401500000$$

$$-0.005613806$$

$$x_6 := -1.401400000$$

$$-0.003785036$$

$$x_7 := -1.401300000$$

$$-0.001956817$$

$$x_8 := -1.401200000$$

$$-0.000129147$$

> **for** i **from** 1 **while** $f\left(x[8] + \frac{i}{1000000}\right) < 0$ **do** $x[i] := x[8] + \frac{i}{1000000}$; $f(x[i])$ **end do**

$$x_1 := -1.401199000$$

$$-0.000110874$$

$$x_2 := -1.401198000$$

$$-0.000092600$$

$$x_3 := -1.401197000$$

$$-0.000074326$$

$$x_4 := -1.401196000$$

$$-0.000056052$$

$$x_5 := -1.401195000$$

$$-0.000037778$$

$$x_6 := -1.401194000$$

$$-0.000019505$$

$$x_7 := -1.401193000$$

$$-0.000001231$$

3.1.3 For two decimals

\>

$y(0) := -1.5;$
$rank := 5:$
for i **from** 0 **while** $f(y(i)) < 0$ **do** $y(i+1) := y(i) + i \cdot 10^{-1}; f(y(i+1))$
end do;
$y(i) := y(0);$
for j **from** 0 **while** $f(y(j)) < 0$ **do** $y(j+1) := y(j) + j \cdot 10^{-2}; f(y(j+1))$
end do;
$y(j);$

$$y(0) := -1.5$$
$$y(1) := -1.5$$
$$-2.09375$$
$$y(2) := -1.400000000$$
$$0.021760000$$
$$y(2) := -1.5$$
$$y(1) := -1.5$$
$$-2.09375$$
$$y(2) := -1.490000000$$
$$-1.853977575$$
$$y(3) := -1.470000000$$
$$-1.394148551$$
$$y(4) := -1.440000000$$
$$-0.751736422$$
$$y(5) := -1.400000000$$
$$0.021760000$$
$$-1.400000000$$

Exercice : write a procedure for calculating a zero of a function

3.2 Numerical resolution of differential equation

3.2.1 Analytic resolution

> $Eq := diff(y(x), x) + y(x) = x$

$$Eq := \frac{d}{dx} y(x) + y(x) = x$$

> $Ics := y(0) = 2$

$$Ics := y(0) = 2$$

> $dsolve(Eq)$

$$y(x) = -1 + x + e^{-x} \_C1$$

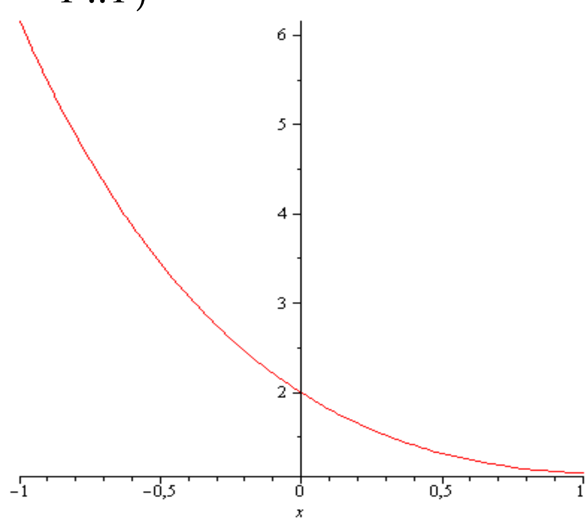
> $dsolve(\{Eq, Ics\})$

$$y(x) = -1 + x + 3\, e^{-x}$$

> $f := (x) \to -1 + x + 3\, e^{-x}$

$$f := x \to -1 + x + 3\, e^{-x}$$

> $plot(f(x), x = -1 ..1)$



3.2.2 Numerical resolution

> $pas := 0.1$

$$pas := 0.1$$

> $x[0] := 0; y[0] := 2$

$$x_0 := 0$$

$$y_0 := 2$$

> $N := 15$

$$N := 15$$

> **for** i **from** 0 **to** N **do** $x[i+1] := x[i]$
> $+ pas$ **end do**

$$x_1 := 0.1$$

$$x_2 := 0.2$$

$$x_3 := 0.3$$

$$x_4 := 0.4$$

$$x_5 := 0.5$$

$$x_6 := 0.6$$

$$x_7 := 0.7$$

$$x_8 := 0.8$$

$$x_9 := 0.9$$

$$x_{10} := 1.0$$

$$x_{11} := 1.1$$

$$x_{12} := 1.2$$

$$x_{13} := 1.3$$

$$x_{14} := 1.4$$

$$x_{15} := 1.5$$

$$x_{16} := 1.6$$

> **for** i **from** 0 **to** N **do** $y[i+1] := (x[i] - y[i]) \cdot pas + y[i]$ **end do**

$$y_1 := 1.8$$

$$y_2 := 1.63$$

$$y_3 := 1.487$$

$$y_4 := 1.3683$$

$$y_5 := 1.27147$$

$$y_6 := 1.194323$$

$$y_7 := 1.1348907$$

$$y_8 := 1.09140163$$

$$y_9 := 1.062261467$$

$$y_{10} := 1.046035320$$

$$y_{11} := 1.041431788$$

$$y_{12} := 1.047288609$$

$$y_{13} := 1.062559748$$

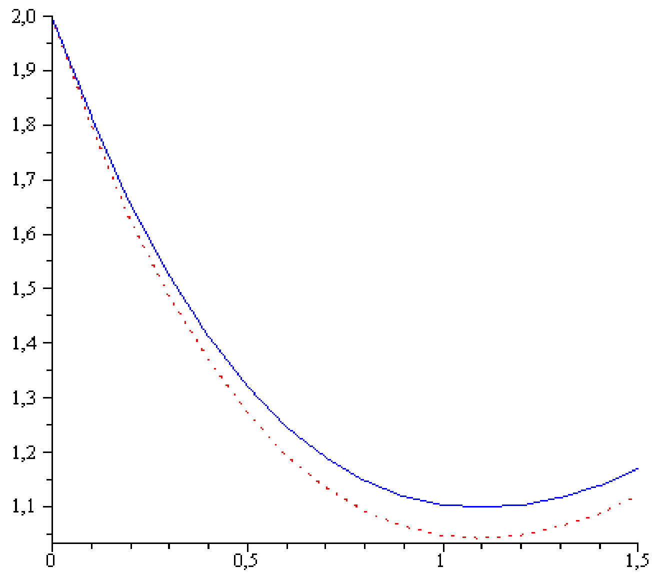
$$y_{14} := 1.086303773$$

$$y_{15} := 1.117673396$$

$$y_{16} := 1.155906056$$

3.2.3 Comparaison

> $plot([\langle\langle seq(x[i], i = 0..N)\rangle | \langle seq(y[i], i = 0..N)\rangle\rangle, \langle\langle seq(x[i], i = 0..N)\rangle | \langle seq(f(x[i]), i = 0..N)\rangle\rangle], linestyle = [dot, solid], color = [red, blue])$


2,0
1,9
1,8
1,7
1,6
1,5
1,4
1,3
1,2
1,1
0
0,5
1
1,5

Références

1. A. J. Marie, A. Hofstetter, Notes de cours Maple, LIRMN, Montpelier, Juin 2002
2. B. Kamoun, Cours Maple, Université de Sfax
3. J. P. Grivet, Méthodes numériques appliquées, Collection Grenoble sciences.
4. Ph. Depondt, La boite à outils de la physique numérique, Cours pour licence de physique L3, Université Pierre et Marie Curie Paris 6 ENS Cachan 2007-2008.
5. H. Klein, Méthodes numériques pour la physique, Licences de physique, faculté des Sciences de Luminy, 13288, Marseille cedex 9